## Torque on a current loop in a uniform magnetic field:

Let us consider a rectangular loop PQRS of length $l$ and breadth $b$ (Fig 1). It carries a current of I along PQRS. The loop is placed in a uniform magnetic field of induction $B$. Let $\theta$ be the angle between the normal to the plane of the loop and the direction of the magnetic field.

The force $\mathrm{F}_{1}$ and $\mathrm{F}_{2}$ are equal in magnitude, opposite in direction and have the same line of action. Hence their resultant effect on the loop is equal.


Fig. 1 Torque on a current loop placed in a magnetic field


Fig. 2 Torque

Force on the arm $\mathrm{QR}, \overrightarrow{\mathrm{F}}_{1}=\overline{\mathrm{I}(\mathrm{QR})} \times \overrightarrow{\mathrm{B}}$
Since the angle between $\overline{\mathrm{I}(\mathrm{QR})}$ and $\overrightarrow{\mathrm{B}}$ is $\left(90^{\circ}-\theta\right)$,
Magnitude of the force $\mathrm{F}_{1}=\mathrm{BIb} \sin \left(90^{\circ}-\theta\right)$
ie. $\quad \mathrm{F}_{1}=\mathrm{BIb} \cos \theta$
Force on the arm SP, $\overrightarrow{\mathrm{F}}_{2}=\overline{\mathrm{I}(\mathrm{SP})} \times \overrightarrow{\mathrm{B}}$
Since the angle between $\overline{\mathrm{I}(\mathrm{SP})}$ and $\overrightarrow{\mathrm{B}}$ is $\left(90^{\circ}+\theta\right)$,
Magnitude of the force $\mathrm{F}_{2}=\mathrm{BIb} \cos \theta$
The forces $F_{1}$ and $F_{2}$ are equal in magnitude, opposite in direction and have the same line of action. Hence their resultant effect on the loop is zero.

Force on the arm PQ, $\vec{F}_{3}=\overline{\mathrm{I}(\mathrm{PQ})} \times \overrightarrow{\mathrm{B}}$
Since the angle between $\overline{\mathrm{I}(\mathrm{PQ})}$ and $\overrightarrow{\mathrm{B}}$ is $90^{\circ}$,

Magnitude of Force $\mathrm{F}_{3}=\operatorname{Bi} l \operatorname{Sin} 90$
$F_{3}$ acts perpendicular to the plane of the paper and outwards.
Force on the arm RS, $\mathrm{F}_{4}=\mathrm{I}(\mathrm{RS}) \times \mathrm{B}$
Since the angle between $\mathrm{I}(\mathrm{RS})$ and is 90 ,
Magnitude of the force $\mathrm{F} 4=\mathrm{BI} l \sin 90=\mathrm{BI} l$

F4 acts perpendicular to the plane of the paper and inwards. The forces F3 and F4 are equal in magnitude, opposite in direction and have different lines of action. So, they constitute a couple.

Hence, Torque $=\mathrm{BI} l \times \mathrm{PN}=\mathrm{BI} l \times \mathrm{PS} \times \sin \theta($ Fig.2)
$=\mathrm{BI} l \times \mathrm{b} \sin \theta=\mathrm{BIA} \sin \theta$

If the coil contains $n$ turns, $\tau=n B I A \sin \theta$

So, the torque is maximum when the coil is parallel to the magnetic field and zero when the coil is perpendicular to the magnetic field.

